Addenda and Corrigenda to

Aperiodic Order

Volume 1: A Mathematical Invitation

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All unmarked corrections below refer to the original printing and have already been included in the electronic version and in the corrected printing of the book. Further corrections are marked by a $*$ on the left margin.

Chapter 2. Preliminaries

The point sets $\varLambda$ in Definition 2.3 and Proposition 2.1 need to be discrete.

$*$ In the proof of Proposition 2.2 on page 16, it should read $F' = F - F$ in the penultimate line.

$*$ On page 18, in the sentence after Definition 2.8, the set $A \subset X$ should be closed.

$*$ In the sentence following Eq. (2.8) on page 24, A_5 is the first *non-Abelian* simple group in the series of alternating groups.

$*$ In the second paragraph of Section 2.3.3, it is tacitly assumed that all group operations are continuous in the topology of G.

Chapter 4. Symbolic Substitutions and Inflations

In the second paragraph of page 82, the set $\mathbb{P}(\mathbb{X})$ is compact in the weak-$*$ topology.

In the last paragraph on page 86, the brief explanation of proximality is, strictly speaking, that of *asymptotic* proximality. This makes no difference for the later use of the term in this volume.

On the right-hand side of the second identity in Eq. (4.14), one should better write $\overline{v_i}$ instead of $\bar{v}_i$. Likewise, in the first equation on page 103, one should write $\overline{v_m}$ instead of $\bar{v}_m$.

$*$ In the proof of Lemma 4.10 on page 106, one needs to use the maximum instead of the minimum for both choices of i.

Chapter 5. Patterns and Tilings

$*$ In Definition 5.1, a pattern is meant to be a *countable* non-empty set of non-empty subsets of $\mathbb{R}^d$. Note that this use of the term 'countable' includes the possibility of a finite set.

One consequence of Definition 5.8 is that the set $\{t \mid \mathcal{T} \sqcap (t+K) = \mathcal{T} \sqcap K\}$ is relatively dense in $\mathbb{R}^d$.

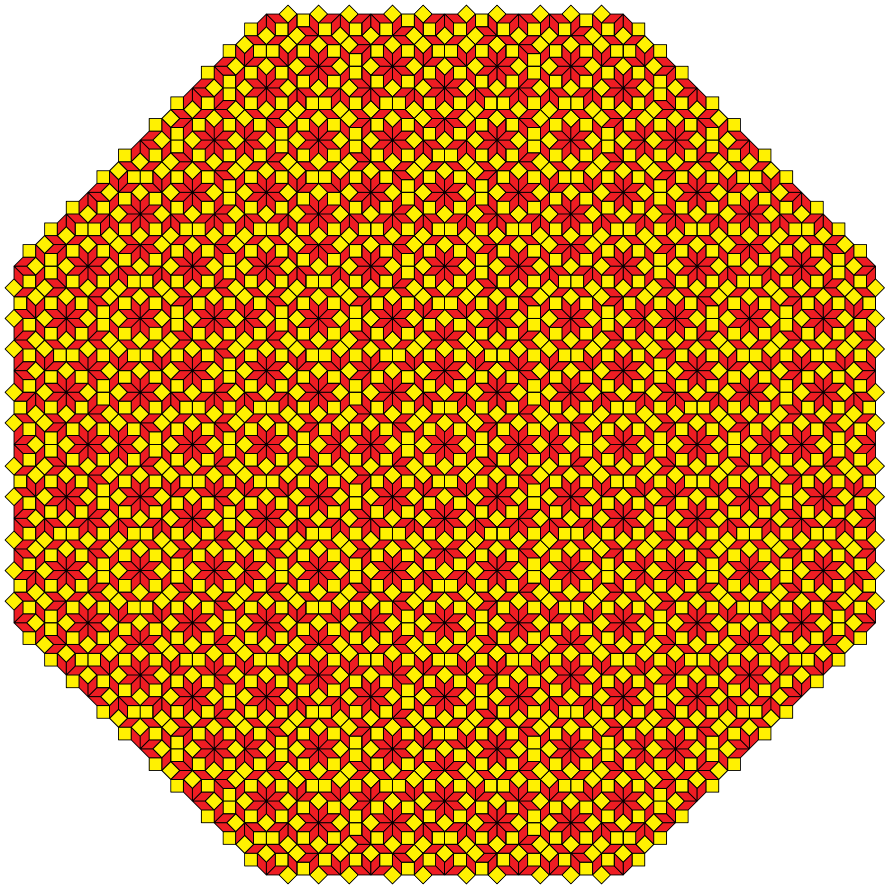

FIGURE 1. The Ammann–Beenker tiling of Figure 6.41 redrawn.

Chapter 6. Inflation Tilings

In Figure 6.41 on page 236 (undecorated Ammann–Beenker tiling), a spurious line appeared in some of the squares in print. The correct figure is reproduced below for convenience.

Chapter 8. Fourier Analysis and Measures

The function f in line 4 of Section 8.1 should also be assumed continuous.

For the explanation of Eq. (8.4), the reader may also want to consult

[Ebe49] Eberlein W.F. (1949). Abstract ergodic theorems and weak almost periodic functions, *Trans. Amer. Math. Soc.* **67**, 217–240.

[Ebe55] Eberlein W.F. (1955). The point spectrum of weakly almost periodic functions, *Michigan Math. J.* **3**, 137–139.

On page 319, three lines before Eq. (8.13), the set S is a Borel set. Similarly, on page 322, the set A in the proof of Lemma 8.2 is Borel.

In line 5 of page 324, the set K must be non-empty.

The proof of Theorem 8.4 also shows that, if measures μ_n norm converge to a measure μ, with $\mu_n \perp \nu$, then also $\mu \perp \nu$.

Chapter 9. Diffraction

Note that the diffraction patterns of Figure 9.2 use the same type of representation as explained in the caption of Figure 9.6.

* In Example 9.8, the amplitude $A(k)$ depends on ε, through the $\star$-map of the dual cut and project scheme.
Eq. (9.22) on page 357 should be completed as

$$L^{\circledast} \;:=\; \pi(\mathcal{L}^*) \;=\; \Big\{ \frac{1}{2}\big(m + \tfrac{n}{\sqrt{2}}\big) \;\big|\; m,n \in \mathbb{Z} \Big\} \;=\; \frac{\sqrt{2}}{4}\mathbb{Z}[\sqrt{2}\,]. \tag{9.22}$$

* The last equation before Corollary 9.4 on page 362 must read

$$\eta(z) \;=\; \mathrm{dens}(\mathcal{L})\,\big(h * \widetilde{h}\big)(z^\star),$$

where $\mathcal{L}$ is the embedding lattice introduced earlier. Consequently, the correct formula for the amplitude in Corollary 9.4 is $A(k) = \mathrm{dens}(\mathcal{L})\,\widehat{h}(-k^\star)$. This is in agreement with Proposition 9.10 and Theorem 9.5 on page 385.

In the last equation of Theorem 9.4 on page 365, the integral should be over W (rather than over H).

In the last sentence of Remark 9.16 on page 379, the relation on the Eberlein convolution von $\delta_{\mathbb{Z}}$ with $\omega_\circ$ can be extended to $\delta_{\mathbb{Z}} \circledast \omega_\circ = \delta_{\mathbb{Z}} \circledast \widetilde{\omega_\circ} = \frac{1}{3}\,\delta_{\mathbb{Z}}$.

Chapter 11. Random Structures

In Example 11.2 on page 436 and in Remark 11.2 on page 439, it might be more adequate to use the term 'measure-theoretic entropy' instead of 'metric entropy'. Both terms are in use in the standard literature.

In Example 11.4, we illustrated two typical configurations of dimers as

$$\ldots[+\;-][-\;+][-\;+][+\;-][-\;+][-\;+][-\;+][+\;-][+\;-]\ldots$$
$$\ldots[-\;+][+\;-][+\;-][-\;+][+\;-][+\;-][+\;-][-\;+][-\;+][+\;-]\ldots$$

On page 441, the reference to Example 4.6 should refer to page 90 rather than to page 89.

In the last equation on page 448, one should write $\mathcal{O}\bigl(|k|^{-1+\varepsilon}\bigr)$.

On page 454, it should read '(symplectically) self-dual' rather than 'symplectic' $N \times N$ matrices.

Appendix B. The Dynamical Spectrum

The function ψ in the third equation on page 487 should be a *bounded* Borel *
function, hence $\psi \in L^{\infty}(\mathbb{T})$. The definition of $\psi(U_S)$ follows via an extension of that for trigonometric polynomials.

References

Ref. [BG13] has appeared, and is now changed to

[BG14] Baake M. and Grimm U. (2014). Squirals and beyond: Substitution tilings with singular continuous spectrum, *Ergodic Th. & Dynam. Syst.* **34**, 1077–1102. `arXiv:1205.1384`.

The following reference has been added in Section 11.4.2:

[BKM14] Baake M., Kösters H. and Moody R.V. (2014). Diffraction theory of point processes: Systems with clumping and repulsion, *Preprint* `arXiv:1405.4255`.

The following reference substantiates the last sentence of Appendix B and has been added.

[BLvE14] Baake M., Lenz D. and van Enter A.C.D. (2014). Dynamical versus diffraction spectrum for structures with finite local complexity, *Ergod. Th. & Dynam. Syst.*, in press. `arXiv:1307.7518`.

Ref. [BO13] has appeared, and has become

[BO14] Barge M. and Olimb C. (2014). Asymptotic structure in substitution tiling spaces, *Erg. Th. & Dynam. Syst.* **34**, 55–94. `arXiv:1101.4902`.

Ref. [BF12] has appeared, and is now changed to

[BF13a] Bédaride N. and Fernique T. (2013). The Ammann-Beenker tilings revisited. In *Aperiodic Crystals*, Schmid S., Withers R.L. and Lifshitz R. (eds.), pp. 59–65 (Springer, Dordrecht). `arXiv:1208.3545`.

The following reference has been added in Section 5.7.2:

[BF13b] Bédaride N. and Fernique T. (2013). When periodicities enforce aperiodicity, *Preprint* `arXiv:1309.3686`.

The following references has been added in Example 10.4:

[CV13] Cellarosi F. and Vinogradov I. (2013). Ergodic properties of k-free integers in number fields, *J. Modern Dynamics* **7**, 461–488. `arXiv:1304.0214`.

The following reference has been added in Remark 5.5:

[CS03] Clark A. and Sadun L. (2003). When size matters: Subshifts and their related tiling spaces, *Ergod. Th. & Dynam. Syst.* **23**, 1043–1057. `arXiv:math.DS/0201152`.

The following reference has been added in Remark 7.1:

[Dub06] Dubickas A. (2006). Arithmetical properties of powers of algebraic numbers, *Bull. London Math. Soc.* **38**, 70–80.

The initials of the first author of Ref. [DV-J88] should read D.J.

Ref. [FS12] has appeared, and is now changed to:

[FS14] Frank N.P. and Sadun L. (2014). Fusion tilings with infinite local complexity, *Top. Proc.* **43**, 235–276. `arXiv:1201.3911`.

Ref. [FR13] has appeared, and is now changed to:

[FR14] Frettlöh D. and Richard C. (2014). Dynamical properties of almost repetitive Delone sets, *Discr. Cont. Dynam. Syst. A* **34**, 531–556. `arXiv:1210.2955`.

Ref. [GJS12] has appeared:

[GJS12] Gähler F., Julien A. and Savinien J. (2012). Combinatorics and topology of the Robinson tiling, *C. R. Math. Acad. Sci. Paris* **350**, 627–631. `arXiv:1203.1387`.

The volume in Ref. [GLA90] should be **85**.

$*$ Ref. [Gou03] should read:

[Gou03] Gouéré J.-B. (2003). Diffraction et mesure de Palm des processus ponctuels (Diffraction and Palm measure of point processes), *C. R. Acad. Sci. Paris, Ser. I* **336**, 57–62. `arXiv:math.PR/0208064`.

The following references has been added in Example 10.4:

[HuB14] Huck C. and Baake M. (2014). Dynamical properties of k-free lattice points *Acta Phys. Pol. A*, in press. `arXiv:1402.2202`.

Ref. [KS12] has appeared, and is now changed to:

[KS14] Kellendonk J. and Sadun L. (2014). Meyer sets, topological eigenvalues, and Cantor fiber bundles, *J. London Math. Soc.* **89**, 114–130. `arXiv:1211.2250`.

In Section 11.2.3, we added the following reference:

[Mol14] Moll M. (2014). Diffraction of random noble means words, *J. Stat. Phys.* **156**, in press. `arXiv:1404.7411`.

Ref. [PH13] has appeared:

[PH13] Pleasants P.A.B. and Huck C. (2013). Entropy and diffraction of the k-free points in n-dimensional lattices, *Discr. Comput. Geom.* **50**, 39–68. `arXiv:1112.1629`.

Ref. [Pow11] has appeared, and is now changed to:

[Pow14] Power S.C. (2014). Crystal frameworks, matrix-valued functions and rigidity operators. In *Concrete Operators, Spectral Theory, Operators in Harmonic Analysis and Approximation*, Cepedello Boiso M., Hedenmalm H., Kaashoek M., Montes Rodríguez A. and Treil S. (eds.), pp. 405–420 (Birkhäuser, Basel). `arXiv:1111.2943`.

Ref. [ST10] was inaccurate, and has been changed to:

[ST09] Siegel A. and Thuswaldner J.M. (2009). *Topological Properties of Rauzy Fractals*, Mém. Soc. Math. France **118** (Société Mathématiques de France, Paris).

Ref. [Ter13] has appeared:

[Ter13] Terauds V. (2013). The inverse problem of pure point diffraction — examples and open questions, *J. Stat. Phys.* **152**, 954–968. `arXiv:1303.3260`.

List of Definitions